Section Notes (9/23)

Peyam Ryan Tabrizian

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1 Midterm 1 Feedback

Just for your information, if I had to grade you just based on this exam, then it would look like this:

- A: Bigger than 38
- B: 30-38
- C: Less than 30

And, for your info, the median score was a 34.

Also, according to Prof. Christ, only worry when you have less than a 24.

Don't worry **too** much about your scores, because:

- **The add/drop deadline is this Friday!** Don't feel ashamed to drop the class if you see it's to overwhelming
- You can still switch your grade to P/NP until the end of the 10th week
- You can still switch to Math 16A before Friday, which is an easier version of Math 1A
- The rest of the semester will be **MUCH** better / more computational!
- This exam only counts for 15 percent of your grade, so you can still get an A if you work hard!

Also, this is Berkeley, where the top students from high schools compete with each other. We just cannot give everyone an A in this class, and just because you get a B it doesn't mean you did poorly on the exam! Remember, you're still a top high school students! If you lower your standards a little bit, you will be much better off! And it **is** possible to do well in life even if you have a couple of Bs! (I had at least 10 B/B+ s in college, but still got into Berkeley!) Motivation is important!

And just if you're wondering, your **exam scores** are not converted to grades. We'll take the numeric value of your exams, and then after the final, add all the points, and then we get a final score. **Only then** we'll rank the students according to your course points!

Also, people who worked hard on their hws did better on the exam, and people who only wrote the bare minimum did not so great! Point is: Spend time doing your hw thoroughly! **Think about the problems**!!!

2 A brief vision into the future

Life goes on, so let's move on!

Math 1A (and 1B) is divided into 3 blocks, and now we've arrived at block 2!

- Block 1: Functions and Limits
- Block 2: Derivatives
- Block 3: Integrals

Good news: No more $\epsilon - \delta$!!!! (except maybe the final)

Now we're able to use all the definitions we learned in Block 1 to do something powerful! (sort of like a Weapon of Math Destruction)

Will learn about derivatives and ways to compute them, and will discover **powerful** applications!

3 Derivatives

Will be **the** most single useful thing we'll learn in this class!

Here, I'll give some motivation: So, suppose you're trying to model a complicated phenomenon (e.g. Weather) with a complicated function. We'd like to **simplify** that function while **preserving** all the info the complicated function gives us.

Now we've seen one way of doing it: Given f and a, approximate $f(x)$ by $f(a)$ for x close to a (e.g. The weather tomorrow will be almost equal to the weather today). This works precisely when f is **continuous**, but doesn't give us a lot of info! In particular, things are usually changing a lot, and approximating sthg by a constant means you're assuming the phenomenon is not really changing.

What's the next easier function you can think of, besides a constant? Yes, a **linear function**! So you try to approximate f at a by a **linear function**. This idea sounds harmless/intuitive, but it is actually **very powerful!!!**. In fact, there's a whole subject devotes to studying linear functions, namely linear algebra (see Math 54).

Now, how would we go about approximating a function f at a by a line? Geometrically, given a graph of a function, we'd like to consider the **tangent line** to the graph of f at a. How the hell could we define that???

First of all, we know that the tangent line must go through the point $(a, f(a))$, so all we need is the **slope** of the tangent line!

For this, use a math-trick (which you'll see over and over and over and over again in your math career): Whenever you don't knwo how to define something, start by defining something you know, and then take some sort of a limit!

In this case, we know how to define slopes of the secant line going through $(a, f(a))$ and $(x, f(x))$:

$$SLOPE = \frac{f(x) - f(a)}{x - a}$$

This is just the definition of the slope seen in precalculus! Now, using the math trick above, just let x get close to a, i.e. take the limit, and you get:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

And we say f is **differentiable** at a whenever this limit exists!

The upshot is: Differentiable functions behave **really** well! Basically, f is differentiable at a if it can be approximated well by a linear function at a.

Non-examples include: $f(x) = |x|$ at 0, or $f(x) = \sqrt{|x|}$ at 0. If you draw their graphs, you'll notice they behave very weirdly at 0.

We also have a physical interpretation: $f'(a)$ is the speed of an object with position f at a. In this case, it is easy to see when functions are not differentiable at a point. Basically, there's an abrupt change in speed at that point! (think of a car breaking really quickly!)

4 The derivative as a function, and higher derivatives

Suppose f is differentiable everywhere.

At **every** point a, can define:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

The upshot is that you can do this for **every** a, hence can define a **new** function g, defined by:

$$g(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

So g inputs a and gives you the slope of the tangent line of f at a. Because the expressions of g and f' look similar, we will also call $g = f'$. f' will be called the **derivative function** (or just the **derivative**) of f.

Be careful, the graph of f' may look very differently from the graph of f, but f' gives us a lot of info about f.

For example, if $f(x) = x^2 + x$, then we can compute:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = 2a + 1$$

This defines a **new** function f' defined by $f'(a) = 2a + 1$. But no reason to use a, we could also have used b or even x! So $f'(x) = 2x + 1$.

Now we can play that game again! Since f' is a function, can define its derivative f'', and so on!!!

For example, for f as above, we get $f'(x) = 2x + 1$, $f''(x) = 2$, etc.

5 Preview: Rules of differentiation

Now how would we actually compute derivatives? It would be very annoying to start from scratch every time!

Our plan is: Start from simple cases, and then build up to more complicated ones!

So for the simple cases, we have:

$$For f(x) = c, f'(x) = 0$$

(c is a constant)

$$For f(x) = x, f'(x) = 1$$

$$For f(x) = x^n, f'(x) = nx^{n-1}$$

(and this works for every nonzero n, whether it be an integer or not)

For more complicated functions, use the following rules:

$$(f + g)'(x) = f'(x) + g'(x)$$

$$(cf)'(x) = cf'(x)$$

(where c is a constant)

However, this rule doesn't apply to fg, i.e. $\boxed{(fg)'(x) \neq f'(x)g'(x)}$
(but we'll see later a similar rule which works!)

Finally, for a little challenge, using the definition of e and the definition of the derivative, try to prove that:

$$(e^x)' = e^x$$